



Applications of complex Huygens principles

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Final Report

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14. ABSTRACT The original goal was to apply the analytic Huygens representations of EM fields (developed jointly with Dr. Thorkild Hansen in 2009-2011) to scattering, communication and radar. However, in 2012 Dr. Richard Albanese urged me to clarify the concept of electromagnetic inertia which I had defined in 2011, and which posed some conceptual problems since it implied that electromagnetic energy in vacuum generally flows at speeds less than c. Thus I changed my goal to understanding EM inertia and relating it to the reactive field energy. But reactive EM energy is strictly linked to the complex Poynting theorem (CPT), which holds only for time-harmonic fields. Even there, no reactive energy density can be defined because this theorem cannot be interpreted as its conservation law. In 2014 I generalized the CPT to EM fields with arbitrary time dependence and showed that an analytic continuation of the generalized CPT does have an interpretation as a conservation law for reactive EM energy. I further defined a complex radiation impedance density analogous to the impedance $Z=R+iX$ of an RLC circuit. I hope to use this analogy to interpret the real and imaginary parts of the complex impedance density as the radiation resistance and radiation reactance densities of the EM field.						
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Final Report
AFOSR Grant # FA9550-12-1-0122
31 March 2012–31 May 2015

Gerald Kaiser
Center for Signals and Waves
Portland, OR
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May 29, 2015

While preparing my grant proposal in 2011, the main objective was to further develop and apply the newly discovered *analytic Huygens wavelet representations* [HK9, HK11, K12a] of electromagnetic fields. One of their main attractions is the potential for *compressing* data, based on the high directivity of these wavelets which makes it possible to ignore components outside any directional cone of interest with minimal loss of accuracy. Dr. Hansen and I believe that such representations may offer a computationally attractive alternative to the Fast Method of Moments. This has, in fact, been the direction of Dr. Hansen's recent work.

My focus changed after discovering [K11a, K11b, K12b] that a generic electromagnetic field (including any field with spatially bounded sources) has a positive *inertia density* throughout its entire space-time region of support. This is a classical version of the well-known fact in QED that virtual photons (i.e., photons which are emitted and/or absorbed by charged matter) have a non-vanishing mass. Only *idealized* photons and EM fields (e.g., traveling plane waves in vacuum) have vanishing inertia, yet this is the basis for the common statement that photons and EM fields are *massless*. Since positive mass is associated with propagation at speeds less than c , the conclusion that EM fields have positive inertia seems to contradict the fact that they propagate in vacuum at speed c . This led to intense and challenging discussions with Dr. Richard Albanese throughout 2011. To clarify the situation, I applied my theory to several concrete examples: standing plane waves and electric or magnetic dipoles with general time dependence. In each case I showed that the local EM *energy* flows at $v < c$, although the *fields* themselves propagate at c as expected. Such discrepancies are due to *interference* between local wave components propagating (at c) in different directions. The clearest example is a standing plane

wave, where the energy executes elastic, singular oscillations between adjacent nodal planes [K11b, Section 4]. The relation between the EM fields and their energy-momentum densities is closely analogous to that between quantum wave functions and their observables, with $\mathbf{F} = \mathbf{E} + i\mathbf{B}$ playing the role of the complex wave function. Observables are *sesquilinear* in the fields (wave functions) and independent of their overall phase, and consequently they obey different equations (conservation laws) than the wave functions (Maxwell's equations).

Aside from the trivial example of traveling plane waves, only very special EM fields have a vanishing inertia density. If a charge-current density can be found whose field has vanishing inertia density, it would be an *ideal radiator*, propagating all of its EM energy at speed c without a near-field and hence without any self-interference. In [K11a] I constructed a class of such solutions which I called *coherent electromagnetic wavelets*. However, the sources of these wavelets consist of a relativistically spinning disk and its entire spin axis, hence they are unbounded and therefore unrealizable. I also wrote a short paper [K12c] proposing *regularized* coherent wavelets as models for *quasars*, where ‘thickened’ versions of the singular source disk and its spin axis represent the accretion disk and the jets of the quasar, respectively. The accretion disk is a thin oblate spheroid, and the jets are thin orthogonal hyperboloids. Although the jets are unbounded, they are a reasonable model for quasar jets since these can extend for billions of light years before being absorbed.

I began to suspect in 2011 that the inertia density of an EM field is related to its *reactive energy* [K11b, K12b]. In the literature, the concept of reactive EM energy is based on the complex Poynting theorem (CPT), which applies only to time-harmonic fields. The *real* and *imaginary* parts of the CPT describe the flows in space and time of *active* and *reactive* energy, respectively.¹ The real part of the CPT can be interpreted as a *conservation law* for the period-averaged active energy, and this gives an expression for the period-averaged active local energy *density* of the field.² However, the imaginary part *cannot* be interpreted as a conservation law, hence no expression can be defined for the local *reactive* energy density. In 2014, I generalized the CPT to EM fields with arbitrary time dependence and showed that an analytic continuation of this generalized CPT does have an interpretation as a conservation law for reactive EM energy.³ Furthermore, I defined a *complex radiation impedance density* $\mathcal{Z} = \mathcal{R} + i\mathcal{X}$ for a general EM field, analogous to the complex impedance $Z = R + iX$ of an RLC circuit. \mathcal{Z} is related to the inertia density \mathcal{I} of the field by $\mathcal{I} = |\mathcal{Z}|$. I hope to use this analogy to interpret \mathcal{R} and \mathcal{X} as the distributed radiation resistance density and radiation reactance density of the EM field, respectively. However, these interpretations are rather speculative at this point and I'm not sure how

¹Active energy can perform work, and reactive energy cannot.

²The ordinary (time-domain) Poynting theorem gives the *instantaneous* active energy density of general fields, while the real part of the CPT gives the *period-averaged* active energy density of time-harmonic fields.

³The reactive energy is conserved in *imaginary time*, which I interpreted as the ‘reactive time’ tracking the lags and leads associated with capacitive and inductive reactive energy.

to substantiate them. For this reason, I have decided not to apply for another grant at this time. I am discussing these ideas with Professor David Griffiths at nearby Read College, and will present my latest results on these topics at the IEEE/URSI Joint Meeting this July 2015 [K15].

Acknowledgements

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- [K12a] G Kaiser, [Huygens' principle in classical electrodynamics: a distributional approach](#). *Advances in Applied Clifford Algebras* Vol. 22, # 3 (2012), Pages 703–720
- [K12b] G Kaiser, [The Reactive Energy of Transient EM Fields](#). IEEE International Symposium on Antennas and Propagation and USNC-URSI National Radio Science Meeting, Session 563, July 13, 2012
- [K12c] G Kaiser, [Electromagnetic helicity wavelets: a model for quasar engines?](#) Preprint, 2012
- [K14] G Kaiser, [Completing the complex Poynting theorem: Conservation of reactive energy in reactive time](#). Preprint, 2014
- [K15] G Kaiser, [Conservation of reactive EM energy in reactive time](#). Invited paper, Special Session on Fundamental Considerations of Electromagnetic Energy and Interactions: Theory and Applications, IEEE AP-S Symposium on Antennas and Propagation and URSI CNC/USNC Joint Meeting, Vancouver, BC, July 2015

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Gerald Kaiser

Program Manager**The AFOSR Program Manager currently assigned to the award**

Dr. Arje Nachman

Reporting Period Start Date

04/01/2012

Reporting Period End Date

05/31/2015

Abstract

The original goal was to apply the analytic Huygens representations of EM fields (developed jointly with Dr. Thorkild Hansen in 2009-2011) to scattering, communication and radar. However, in 2012 Dr. Richard Albanese urged me to clarify the concept of electromagnetic inertia which I had defined in 2011, and which posed some conceptual problems since it implied that electromagnetic energy in vacuum generally flows at speeds less than c . Thus I changed my goal to understanding EM inertia and relating it to the reactive field energy. But reactive EM energy is strictly linked to the complex Poynting theorem (CPT), which holds only for time-harmonic fields. Even there, no reactive energy density can be defined because this theorem cannot be interpreted as its conservation law. In 2014 I generalized the CPT to EM fields with arbitrary time dependence and showed that an analytic continuation of the generalized CPT does have an interpretation as a conservation law for reactive EM energy. I further defined a complex radiation impedance density analogous to the impedance $Z=R+iX$ of an RLC circuit. I hope to use this analogy to interpret the real and imaginary parts of the complex impedance density as the radiation resistance and radiation reactance densities of a general EM field.

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2012: G. Kaiser, "Huygens' principle in classical electrodynamics: a distributional approach." Advances in Applied Clifford Algebras, Vol. 22, # 3

(2012), Pages 703-720. <http://arxiv.org/abs/0906.4167>

2012: G. Kaiser, "Electromagnetic helicity wavelets: a model for quasar engines?" Preprint, 2012.

<http://arxiv.org/abs/1209.2913>

2014: G. Kaiser, "Completing the complex Poynting theorem: Conservation of reactive energy in reactive time." Preprint, 2014. <http://arxiv.org/abs/1412.3850>

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<http://arxiv.org/abs/1501.01005>

Changes in research objectives (if any):

The original goal was to apply the analytic Huygens wavelet representations of acoustic and electromagnetic fields developed in 2009-2011 with Dr. Thorkild Hansen. Then Dr. Richard Albanese suggested in 2012 that I change the goal to clarifying the properties of the EM inertia density I had discovered in 2011. This led in 2014 to a generalization of the complex Poynting theorem from time-harmonic to arbitrary EM fields and its extension to complex time. That in turn led to a definition of the complex radiation impedance density of a general EM field, whose real and imaginary parts I believe to be related to the radiation resistance and radiation reactance densities of the field.

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